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Mathematical Induction

Date__________________ Period____

Write the induction proof statements P_1, P_k, and P_{k+1} for each conjecture.

1) P_n: $11 + 19 + 27 + \cdots + 8n + 3 = n(4n + 7)$

2) P_n: $7^n - 4^n$ is divisible by 3

3) P_n: $4^n \geq 4n$

Use mathematical induction to prove that each statement is true for all positive integers

4) $36 + 324 + 900 + \cdots + (12n - 6)^2 = 12n(4n^2 - 1)$

5) 3 is a factor of $4^n + 2$

6) $7^n \geq 7n$

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Mathematical Induction

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Write the induction proof statements P_1, P_k, and P_{k+1} for each conjecture.

1) P_n: $11 + 19 + 27 + \cdots + 8n + 3 = n(4n + 7)$

P_1: $8 + 3 = 4 + 7$

P_k: $11 + 19 + 27 + \cdots + 8k + 3 = k(4k + 7)$

P_{k+1}: $11 + 19 + 27 + \cdots + 8k + 3 + 8(k + 1) + 3 = k(4k + 7) + 8(k + 1) + 3$

2) P_n: $7^n - 4^n$ is divisible by 3

P_1: $7^1 - 4^1$ is divisible by 3

P_k: $7^k - 4^k$ is divisible by 3. Therefore, $7^k - 4^k = 3r$ for some integer r.

P_{k+1}: $7^{(k+1)} - 4^{(k+1)}$ is divisible by 3

3) P_n: $4^n \geq 4n$

P_1: $4^1 \geq 4 \cdot 1$

P_k: $4^k \geq 4k$ for some positive integer $k \geq 1$

P_{k+1}: $4^{k+1} \geq 4(k + 1)$

Use mathematical induction to prove that each statement is true for all positive integers

4) $36 + 324 + 900 + \cdots + (12n - 6)^2 = 12n(4n^2 - 1)$

Let P_n be the statement $36 + 324 + 900 + \cdots + (12n - 6)^2 = 12n(4n^2 - 1)$

Anchor Step

P_1 is true since $(12 - 6)^2 = 12(4 \cdot 1^2 - 1)$

Inductive Hypothesis

Assume that P_k is true: $36 + 324 + 900 + \cdots + (12k - 6)^2 = 12k(4k^2 - 1)$

Inductive Step

We now show that P_{k+1} is true:

$$36 + 324 + 900 + \cdots + (12k - 6)^2 + (12(k + 1) - 6)^2 = 12k(4k^2 - 1) + (12(k + 1) - 6)^2$$
$$48k^3 - 12k + (12k - 6)^2$$
$$48k^3 - 12k + 144k^2 + 144k + 36$$
$$48k^3 + 144k^2 + 132k + 36$$
$$12(4k^3 + 12k^2 + 11k + 3)$$
$$12(4k^3 + 12k^2 + 12k + 4 - k - 1)$$
$$12(4(k^3 + 3k^2 + 3k + 1) - (k + 1))$$
$$12(4(k + 1)^3 - (k + 1))$$
$$12(k + 1)(4(k + 1)^2 - 1)$$

Conclusion

By induction P_n is true for all $n \geq 1$.

5) 3 is a factor of $4^n + 2$

Let P_n be the statement 3 is a factor of $4^n + 2$

Anchor Step

P_1 is true: 3 is a factor of $4^1 + 2$

Inductive Hypothesis

Assume that 3 is a factor of $4^k + 2$. Therefore, $4^k + 2 = 3r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: 3 is a factor of $4^{(k+1)} + 2$

$4 \cdot 4^k + 2$

$(3 + 1) \cdot 4^k + 2$

$3 \cdot 4^k + 4^k + 2$

$3 \cdot 4^k + 3r$

$3(4^k + r)$

Conclusion

By induction P_n is true for all $n \geq 1$.

6) $7^n \geq 7n$

Let P_n be the statement $7^n \geq 7n$

Anchor Step

P_1 is true: $7^1 \geq 7 \cdot 1$

Inductive Hypothesis

Assume that P_k is true: $7^k \geq 7k$ for some positive integer $k \geq 1$

Inductive Step

We now show that P_{k+1} is true:

$7^k \geq 7k$ $\quad$ $k \geq 1$

$7 \cdot 7^k \geq 7 \cdot 7k$ $\quad$ $42k \geq 42 \geq 7$

$7^{k+1} \geq 49k$

$7^{k+1} \geq 7k + 42k$

$7^{k+1} \geq 7k + 7$

$7^{k+1} \geq 7(k + 1)$

Conclusion

By induction P_n is true for all $n \geq 1$.

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